CS-C2150 Theoretical Computer Science   Spring 2019
Exam Wed 10 April 2019, 5–8 p.m.

Use of calculators is not allowed in the exam.

You are allowed to bring with you a single one-sided A4 "cheat sheet", **personally handwritten by you. (NO photocopies, NO printouts, NO computer type-set text.)** Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.

Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. Show that each of the following languages is regular, by describing it either in terms of a regular expression or in terms of a (deterministic or nondeterministic) finite automaton:

   (a) $\{w \in \{0,1\}^* \mid w$ contains three consequent zeros or three consequent ones (or both)$\}$
   (b) $\{w \in \{0,1\}^* \mid w$ contains neither three consequent zeros nor three consequent ones$\}$
   (c) $\{w \in \{0,1\}^* \mid$ the number of ones in w is a multiple of three (possibly zero)$\}$
   (d) $\{w \in \{0,1\}^* \mid |w| \geq 3$ and the third-to-last symbol in w is a 1$\}$

   20 points

2. Consider the following context-free grammar G:

   $$S \rightarrow aSB \mid \varepsilon$$
   $$B \rightarrow bB \mid b$$

   (a) Describe the language generated by G in plain English, as simply as you can. (3 pts)
   (b) Show that G is ambiguous. (5 pts)
   (c) Design an unambiguous grammar G' that generates the same language as G. (5 pts)
   (d) Prove (precisely!) that the language generated by G is not regular. (7 pts)

   20 points

3. A *party walk* is a sequence of consequent steps, whose direction with respect to the starting point is either forward (abbr. f), backward (b), left (l) or right (r). For instance, the sequence *flbbrrfrff* describes the following walk, whose total result is to move the walker a distance of two steps forward (and concurrently two steps to the right):
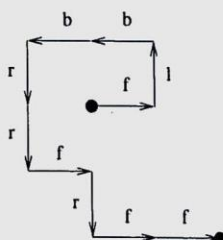
   Design a context-free grammar that generates all party walks whose total result is to move the walker at least one step forward from the starting point (ignoring any possible sideways movement). Present a parse tree for the example sequence *flbbrrfrff* in your grammar.

   10 points

4. Design a deterministic single-tape Turing machine that checks that the binary string it receives as input contains at least as many ones as zeros. (Present the machine preferably in the form of a state diagram rather than a transition table.) Show the accepting computation sequence ("run") of your machine on input 1001, and the rejecting computation sequence on input 010.

   10 points